Alternative Form of the Parity-Violating Current for the Hyperon Weak Radiative Decays and Hara Theorem

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Abstract

It is shown that upon considering an alternative form of a parity-violating part of the transition electromagnetic current it is possible to reformulate Hara theorem in a way that it does not forbid any more nonzero asymmetry in the hyperon weak radiative decays $\Sigma^+ \to p + \gamma$ and $\Xi^- \to \Sigma^- + \gamma$ in the limit of exact $SU(3)_f$ symmetry thus resolving a contradiction with the data and maybe revealing hitherto unseen transition toroid dipole moments. A result is consistent with the traditional one on the single-quark weak radiative transition models. We have also reproduced Vasanti formula at the quark level. As for two-quark weak radiative transitions we have found that the important part of it contains also a toroid dipole moment contribution which seems to be an intrinsic reason of the apparent contradiction between the Hara theorem conclusion and quark model results for hyperon weak radiative decays.

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1. INTRODUCTION

The weak radiative decays have been first analyzed theoretically about forty years ago [1]- [3]. It was envisaged already in [4] a possibility to understand it in the framework of pole model similar to the nonleptonic hyperon decays. Even earlier estimations of the decay rates were made basing on the pion photoproduction amplitudes [5]. At the same time two experiments were performed [6,7], where the first events of the decay $\Sigma^+ \to p + \gamma$ were found (in all 7 events). Unitary symmetry arrived, more elaborated schemes appeared (see, e.g., [8]) and a theorem was proved by Hara that decay asymmetry in the charged hyperon weak radiative decays $\Sigma^+ \to p + \gamma$ and $\Xi^- \to \Sigma^- + \gamma$ should vanish in the limit

of exact $SU(3)_f$ [9] while it can be non zero for the neutral decays $(\Sigma^0, \Lambda) \Rightarrow n + \gamma$ and $\Xi^0 \rightarrow (\Sigma^0, \Lambda) + \gamma$. Since experimental discovery of a large negative asymmetry in the radiative decay $\Sigma^+ \rightarrow p + \gamma$ [10], confirmed later [11] (see the Table 1), the explanation of the net contradiction between experimental results and the Hara theorem prediction "has constituted a constant challenge to theorists" [12]. The contradiction looks even more strange in the light of existing estimates for asymmetry values of other hyperon radiative decays which though not measured so precise as that of $\Sigma^+ \rightarrow p + \gamma$ seem to be of the same order of magnitude [11]. It would eventually require large $SU(3)_f$ symmetry breaking terms but they hardly can be large enough due to the well-known Ademollo-Gatto theorem [13] in order to be able to account for this difficulty.

Another puzzle is related to inconsistency between $SU(3)_f$ symmetry and quark model predictions for the asymmetry value in the hyperon weak radiative decays. Indeed, the Hara theorem was formulated at the hadron level in terms of the $SU(3)_f$ baryon wave functions. It would seem natural if in the framework of a quark model one arrived at a similar result with some minor deviations. But quark models while more or less succeding in describing experimental data on branching ratios and asymmetry parameters (see, e.g., a review [12] and citations therein) did not reproduce the Hara claim without making vanish all asymmetry parameters in the $SU(3)_f$ symmetry limit. The origin of this discrepancy is not clear up to now although many authors have investigated this problem thoroughly [14]- [18] (see also [12] for very complete list of publications) and is a real puzzle as similar calculations say of baryon magnetic moments are known to be rather consistent at the quark and hadron level. A single quark weak radiative transition $s \rightarrow d + \gamma$ was calculated in the framework of the standard Weinberg-Salam model with diagram technique [19], and it was shown that its decay asymmetry is proportional to the ratio $(m_s - m_d)/(m_s + m_d)$ where $m_q\,(q = d, s)$ is the current quark mass. The same result was obtained earlier in [20] on invoking chiral symmetry arguments. So the parity-violating quark transition amplitude goes to zero in the chiral symmetry limit. But these results cannot be, generally speaking,

immediately translated to the hyperon weak radiative decays, as one should first relate current quark evaluations to baryon picture which is far from trivial. But the single-quark contributions proved to be too small to account for the observed decay rates. Even penguin diagram contributions are not strong enough to enhance an effective $s \to d + \gamma$. Two-quark weak radiative diagrams (those with exchange of W-boson between two quarks, one of them emitting a photon, the third quark being a spectator) were shown to give in general nonzero contribution to decay asymmetry of all 5 hyperon radiative decays including $\Sigma^+ \to p+\gamma$ [14]-[16]. (Note that the 6th decay $\Xi^- \to \Sigma^- + \gamma$ cannot proceed via the two-quark W-exchange diagram). Moreover their contribution into the decay rate proved to be important. But the problem of the mutual inconsistency between two-quark diagram results and Hara theorem prediction still persists.

In what follows we shall try to show that a discrepancy between the Hara theorem prediction and experimental result for the asymmetry in the decay $\Sigma^+ \to p + \gamma$ may be overcome due to alternative possibility of multipole parametrization of the axial transition electromagnetic current which include not only dipole transition moment but also contribution of the toroid dipole moment [21,22]. As it has been pointed recently [23], toroid dipole moment naturally arrives in the parity-violating (PV) part of the transition weak radiative matrix element and leads to reformulation of the Hara theorem. We shall also show that the Vasanti result as to the single-quark weak radiative transition [20] is reproduced in our scheme while going from hadron to quark level. Two-quark weak radiative transition are considered also and a source of discrepancy between the results of $SU(3)$ symmetry approach of [9] and that of quark models [14,15] and others seems to be established.

2. KINEMATICS OF THE HYPERON RADIATIVE DECAY PROCESS

The two-body radiative decay amplitude $A(B_1 \to B_2+\gamma)$ in the rest frame of B , usually is written in the form [1]

$$A(B_1 \to B_2\gamma) = i\sqrt{\frac{m_N}{4\pi k_0 E_N}}\overline{u}_2(p_2)(C + D\gamma_5)\sigma_{\mu\nu}k^\nu u_1(p_1)\delta^4(p_1 - p_2 - k), \tag{2.1}$$

which preserves automatically gauge invariance condition. Here $u_{1,2}$ are Dirac spinors of the baryons $B_{1,2}$ with masses $m_{1,2}$, respectively, while C and D are parity-conserving (PC) and parity-violating (PV) amplitudes, correspondingly. The photon momentum value k_ν is entirely determined in the rest frame of B by baryon masses $m_{1,2}$, The angular distribution of the photon reads

$$W(\theta) = \frac{1}{16\pi}\left(\frac{m_1^2 - m_2^2}{m_1}\right)^3 (|C|^2 + |D|^2)[1 + \alpha(\widehat{s}\widehat{p})],$$

where $\widehat{s}$ is the polarization vector of B in its rest frame, θ is the angle between the direction of polarization of B and the momentum of B', and $\widehat{p}$ being the direction of the momentum of B', while α is the decay asymmery parameter. In terms of C and D the decay asymmetry is written as [1]

$$\alpha = \frac{2Re(C^* D)}{|C|^2 + |D|^2}. \tag{2.2}$$

The corresponding decay rate is given by

$$R = \frac{1}{8\pi}\left(\frac{m_1^2 - m_2^2}{m_1}\right)^3 (|C|^2 + |D|^2). \tag{2.3}$$

Experimental data on decay rates in the form of corresponding branching ratios $BR = R(B \to B' + \gamma)/R(total))$ and asymmetry parameters of the relevant decays [11] are placed in the Table 1.

3. TOWARDS THE MULTIPOLE PARAMETRIZATION OF TRANSITION VECTOR CURRENT

The transition vector current of the two particles with spin 1/2 and parity can be expressed in terms of 5 Lorentz structures γ_μ, P_μ, k_μ, $\sigma_{\mu\nu}k^\nu$ and $\sigma_{\mu\nu}P^\nu$, where $P_\mu = (p_1 + p_2)_\mu$, $k_\mu = (p_1 - p_2)_\mu$, $\sigma_{\mu\nu} = (i/2)\,[\gamma_\mu,\, \gamma_\nu]$. Upon using current conservation condition one is left with two Lorentz structures, so the effective transition parity-conserving current reads in one of the forms [21]:

$$J^{(V)}_\mu(k_\nu) = \frac{e\eta}{(2\pi)^3}\overline{u}_2\left[\frac{1}{M^2}\left(k^2_\lambda\gamma_\mu - \widehat{k}k_\mu\right)F_1(k^2_\lambda) + \frac{1}{M}\sigma_{\mu\nu}k_\mu F_2(k^2_\lambda)\right]u_1 \tag{3.1}$$

$$= \frac{e\eta\gamma^2}{(2\pi)^3}\overline{u}_2\left[\frac{1}{M^3}\left(k^2_\lambda P_\mu - (k_\nu P_\nu)k_\mu\right)F_3(k^2_\lambda) + \frac{1}{\gamma^2 M}\sigma_{\mu\nu}k_\mu F_4(k^2_\lambda)\right]u_1 \tag{3.2}$$

$$= \frac{e\eta\gamma^2}{(2\pi)^3}\overline{u}_2\left[\frac{1}{M^3}\left(k^2_\lambda P_\mu - (k_\nu P_\nu)k_\mu\right)F_5(k^2_\lambda) + \frac{1}{\gamma^2 M^2}(k^2_\lambda\gamma_\mu - \widehat{k}k_\mu)F_6(k^2_\lambda)\right]u_1 \tag{3.3}$$

$$= \frac{e\eta\gamma^2}{(2\pi)^3}\overline{u}_2\left[\frac{1}{M^3}\left(k^2_\lambda P_\mu - (k_\nu P_\nu)k_\mu\right)F^{(e)}(k^2_\lambda) + \frac{i}{M^2}\epsilon_{\mu\nu\lambda\sigma}P_\nu k_\lambda\gamma_\sigma\gamma_5 F^{(m)}(k^2_\lambda)\right]u_1. \tag{3.4}$$

Here $M = m_1 + m_2$, $\Delta m = m_1 - m_2$, $\eta = \sqrt{1 - \Delta m^2/M^2}$ and $\gamma = 1/\sqrt{1 - k^2_\lambda/M^2}$ is the Lorentz-factor.

The form factors of each current configuration can be expressed in terms of another one upon using Gordon identities in the form [21,24] (only two of them are independent):

$$\overline{u}_2\left\{k^2_\lambda\sigma_{\mu\nu}k_\nu + M(k^2_\lambda\gamma_\mu - \widehat{k}k_\mu) - \left[k^2_\lambda P_\mu - (k_\nu P_\nu)k_\mu\right]\right\}u_1 = 0,$$

$$\overline{u}_2\left\{ik^2\epsilon_{\mu\nu\lambda\sigma}P_\nu k_\lambda\gamma_\sigma\gamma_5 + M\left[k^2_\lambda P_\mu - (k_\nu P_\nu)k_\mu\right] + (k^2_\lambda - M^2)(k^2_\lambda\gamma_\mu - \widehat{k}k_\mu)\right\}u_1 = 0,$$

$$\overline{u}_2\left\{(M^2 - k^2_\lambda)\sigma_{\mu\nu}k_\nu + iM\epsilon_{\mu\nu\lambda\sigma}P_\nu k_\lambda\gamma_\sigma\gamma_5 + \left[k^2_\lambda P_\mu - (k_\nu P_\nu)k_\mu\right]\right\}u_1 = 0,$$

$$\overline{u}_2\left\{i\epsilon_{\mu\nu\lambda\sigma}P_\nu k_\lambda\gamma_\sigma\gamma_5 + M\sigma_{\mu\nu}k_\nu + (k^2_\lambda\gamma_\mu - \widehat{k}k_\mu)\right\}u_1 = 0.$$

Using the identities we may find out relations between all parametrizations Eqs.(3.1)-(3.4) and observe some kinematic peculiarities. Moreover, pursuing the multipole analysis of currents in spirit of [26,25] one can see that in general form factors do not correspond to the definite multipole distributions. Only those of Eq.(3.4) are in fact multipole ones and may be classified on the complete scheme of multipole expansion of classical electromagnetic current [21,24]. The full formalism of multipole consideration includes not only moments but, in principle, an infinite sequence of $2n$-power radius for each moment. Generally, the latter corresponds to expansion of each form-factors in series on $\boldsymbol{k}^2$ [21]. However, considering two-body decay the form-factor identification with the parameters mentioned may be done only in a special reference system, named in [21] by the intrinsic one (see also [24]). This reference system is given by the equality of kinetic energies (e.k.e.) of both the baryons involved and enables us to write the nonrelativistic reduction of Eq.(3.4)

- for the electric contribution

$$
\begin{aligned}
\frac{\eta\gamma^2}{M}F^{(e)}(k_\lambda^2) \quad \overline{u}_2 \quad & \left(k_\lambda^2 P_\mu - (k_\nu P_\nu)k_\mu\right) u_1 A_\mu \\
& \stackrel{e.k.e.}{\Longrightarrow} F^{(e)}(\Delta m^2 - \boldsymbol{k}^2)\varphi_2^+\varphi_1 \left(-\boldsymbol{k}^2\Phi - \Delta m \boldsymbol{k}\boldsymbol{A}\right) \\
& \Longrightarrow F^{(e)}(\Delta m^2 - \boldsymbol{k}^2)\varphi_2^+\varphi_1 \left(\boldsymbol{\nabla}^2\Phi + \boldsymbol{\nabla}\dot{\boldsymbol{A}}\right) \\
& \stackrel{|\boldsymbol{k}|\to 0}{\Longrightarrow} -F^{(e)}(\Delta m^2)\varphi_2^+\varphi_1 \mathrm{div}\boldsymbol{E} =: -\frac{1}{6}\overline{r_Q^2}\varphi_2^+\varphi_1\rho^{ext},
\end{aligned} \tag{3.5}
$$

- for the magnetic contribution

$$
\begin{aligned}
\frac{i\eta\gamma^2}{M}F^{(m)}(k_\lambda^2) \quad \overline{u}_2 \quad & \epsilon_{\mu\nu\lambda\rho}P_\nu k_\lambda \gamma_\rho\gamma_5 u_1 A_\mu \\
& \stackrel{e.k.e.}{\Longrightarrow} \frac{i\gamma}{M}F^{(m)}(\Delta m^2 - \boldsymbol{k}^2)\varphi_2^+ \left(\epsilon_{iojk}P_0 k_j \sigma_k + \epsilon_{ij0k}k_0\sigma_k\right)\varphi_1 A_i \\
& = \quad iF^m(\Delta m^2 - \boldsymbol{k}^2)\varphi_2^+ \left(-\frac{\boldsymbol{k}^2[\boldsymbol{k}\times\boldsymbol{\sigma}]}{k_\lambda^2} + \frac{\Delta m^2[\boldsymbol{k}\times\boldsymbol{\sigma}]}{k_\lambda^2}\right)\varphi_1\boldsymbol{A} \\
& \stackrel{|\boldsymbol{k}|\to 0}{\Longrightarrow} F^{(m)}(\Delta m^2)\varphi_2^+\boldsymbol{\sigma}\varphi_1 \mathrm{rot}\boldsymbol{A} =: \boldsymbol{\mu}\boldsymbol{B}.
\end{aligned} \tag{3.6}
$$

Here Φ and $\boldsymbol{A}$ are the external potentials, $\boldsymbol{E}$ and $\boldsymbol{B}$ are the electric and magnetic fields, respectively, and $\varphi_{1,2}$ are Pauli spinors of baryons in consideration.

Thus we may connect $F^{(e)}(\Delta m^2)$ with the standard multipole parameter $\overline{r_Q^2}$, the mean-square radius of charge density distribution, and consider $F^{(m)}(\Delta m^2)$ as the projection of magnetic dipole moment on $\boldsymbol{\sigma}$. Taking into account the relation between our parametrizations easily to find the usual decomposition for the diagonal case $m_1 = m_2 = m_0$, e.g.

$$
\overline{r_q^2} = \frac{3e}{2m_0^2}F^{(e)}(\Delta m^2) = \frac{3e}{2m_0^2}\left[F_5(0) + F_6(0)\right],
$$

where $\overline{r_q^2}$ of the baryon considered is given in e/m_0^2 unit that corresponds to the normalization factors in Eqs.(3.1)-(3.4). Instead other parametrizations do not give such simple answer. It is not a proof of the validity of this very expansion given by Eq.(3.4). So, more natural to use the multipole parametrization for the vector current. But the situation is more dramatic in the case of the axial-vector current as it will be seen in the next section.

4. TOWARDS THE MULTIPOLE PARAMETRIZATION OF TRANSITION AXIAL CURRENT

Let us consider axial electromagnetic transition current of the two particles with spin 1/2 and parity. Its possible form is not unique as the most general expression can be written in terms of 5 Lorentz structures: $\gamma_\mu\gamma_5$, $P_\mu\gamma_5$, $k_\mu\gamma_5$, $\sigma_{\mu\nu}k^\nu\gamma_5$ and $i\epsilon_{\mu\nu\rho\lambda}\gamma_\nu P_\rho k_\lambda$, where $P_\mu = (p_1+p_2)_\mu$, $k_\mu = (p_1-p_2)_\mu$, $\sigma_{\mu\nu} = (i/2)\,[\gamma_\mu,\gamma_\nu]$. But due to electromagnetic current conservation and generalized Gordon identities (see, e.g., [21]) (only two of them are independent)

$$\overline{u}_2\{\Delta m\sigma_{\mu\nu}k_\nu + (k_\lambda^2\gamma_\mu - \widehat{k}k_\mu) + i\varepsilon_{\mu\nu\lambda\sigma}P_\nu k_\lambda\gamma_\sigma\gamma_5\}\gamma_5 u_1 = 0,$$

$$\overline{u}_2\{k_\lambda^2\sigma_{\mu\nu}k_\nu + \Delta m(k_\lambda^2\gamma_\mu - \widehat{k}k_\mu) + [k_\lambda^2 P_\mu - k_\lambda P_\lambda k_\mu]\}\gamma_5 u_1 = 0,$$

$$\overline{u}_2\{ik^2\varepsilon_{\mu\nu\lambda\sigma}P_\nu k_\lambda\gamma_\sigma\gamma_5 + (\Delta m^2 - k^2)(k_\lambda^2\gamma_\mu - \widehat{k}k_\mu) + \Delta m[k_\lambda^2 P_\mu - k_\lambda P_\lambda k_\mu]\}\gamma_5 u_1 = 0,$$

$$\overline{u}_2\{-i\Delta m\varepsilon_{\mu\nu\lambda\sigma}P_\nu k_\lambda\gamma_\sigma\gamma_5 + (k^2 - \Delta m^2)\sigma_{\mu\nu}k_\nu + [k_\lambda^2 P_\mu - k_\lambda P_\lambda k_\mu]\}\gamma_5 u_1 = 0,$$

where $\Delta m = m_1 - m_2$ and $u_{1,2}$ are the Dirac spinors of baryons with masses $m_{1,2}$, this transition current can be reduced to one of the following forms [21,22]

$$J_\mu^{(A)}(k_\nu) = \frac{e\eta\gamma}{(2\pi)^3}\overline{u}_2\left[\frac{1}{M^2}(k_\lambda^2\gamma_\mu - kk_\mu)G_1(k_\lambda^2) + \frac{1}{M}\sigma_{\mu\nu}k_\mu G_2(k_\lambda^2)\right]\gamma_5 u_1, \tag{4.1}$$

$$J_\mu^{(A)}(k_\nu) = \frac{e\eta\gamma}{(2\pi)^3}\overline{u}_2\left[\frac{1}{M}\sigma_{\mu\nu}k_\mu G^{(d)}(\Delta m^2) + \frac{k_\lambda^2 P_\mu - (k_\nu P_\nu)k_\mu}{M^3(k_\lambda^2 - \Delta m^2)}[G^{(d)}(k_\lambda^2) - \right.$$
$$\left. - G^{(d)}(\Delta m^2)] + \frac{i}{M^2}\epsilon_{\mu\nu\lambda\sigma}P_\nu k_\lambda\gamma_\sigma\gamma_5 G^{(T)}(k_\lambda^2)\right]\gamma_5 u_1, \tag{4.2}$$

where the kinematic notations are the same as in Sect.3.

Remark that the form factors introduced by Eq.(4.1) do not correspond to the well-defined multipole expansion of currents [21]- [26]. That is why we would like to base our discussion on the Eq.(4.2) which, as has been shown explicitly in [24], does correspond to a definite multipole expansion in a properly chosen reference system, where $k_\mu^2 = \Delta m^2 - \boldsymbol{k}^2$. In this reference system the nonrelativistic reduction of Eq.(4.2) has the forms [24]

- for electric contribution

$$
\begin{aligned}
G^{(d)}(k_\mu^2)\overline{u}_2 i\sigma_{\mu\nu}k_\nu\gamma_5 u_1 A_\mu &\stackrel{e.k.e.}{\Longrightarrow} G^{(d)}(\Delta m^2-\boldsymbol{k}^2)\varphi_2^+\boldsymbol{\sigma}\varphi_1[i\boldsymbol{k}\Phi+i\Delta m\boldsymbol{A}] \\
&\Longrightarrow G^{(d)}(\Delta m^2-\boldsymbol{k}^2)\varphi_2^+\boldsymbol{\sigma}\varphi_1[\boldsymbol{\nabla}\Phi+\dot{\boldsymbol{A}}] \\
&\stackrel{|\boldsymbol{k}|\to 0}{\Longrightarrow} -G^{(d)}(\Delta m^2)\varphi_2^+\boldsymbol{\sigma}\varphi_1\boldsymbol{E} \\
&=: \;\; -\boldsymbol{dE},
\end{aligned}
\tag{4.3}
$$

- for toroid contribution

$$
\begin{aligned}
-G^{(T)}(k_\mu^2)\overline{u}_2 i\epsilon_{\mu\nu\rho\lambda}\gamma_\nu P_\rho k_\lambda u_1 A_\mu &\stackrel{e.k.e.}{\Longrightarrow} -G^{(T)}(\Delta m^2-\boldsymbol{k}^2)\boldsymbol{k}\times[\boldsymbol{k}\times\boldsymbol{\sigma}]\boldsymbol{A} \\
&\Longrightarrow G^{(T)}(\Delta m^2-\boldsymbol{k}^2)\varphi_2^+\boldsymbol{\sigma}\varphi_1\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\boldsymbol{A} \\
&\stackrel{|\boldsymbol{k}|\to 0}{\Longrightarrow} G^{(T)}(\Delta m^2)\varphi_2^+\boldsymbol{\sigma}\varphi_1\boldsymbol{\nabla}\times\boldsymbol{B} \\
&=: \;\; \boldsymbol{T}\ \mathrm{rot}\boldsymbol{B}.
\end{aligned}
\tag{4.4}
$$

Here $\boldsymbol{d}$ and $\boldsymbol{T}$ are the electric and toroid dipole moments [21,22].

One can see that indeed the parametrization given by the Eq.(4.2) is a multipole one where the projections of electric and toroid transition dipole moments are given, respectively, by

$$
d = (e/M)G^{(d)}(\Delta m^2), \tag{4.5}
$$

$$
T = (e/M^2)G^{(T)}(\Delta m^2). \tag{4.6}
$$

We remind that the toroid dipole moment transition violates parity but not T-invariance while the electrical dipole violates both P- and T-invariance. The derivatives of formfactors $G^{(d)}(k_\lambda^2)$ and $G^{(T)}(k_\lambda^2)$ define the corresponding transition averaged radii. Since

$$
G_2(k_\lambda^2) = G^{(d)}(\Delta m^2) + \frac{k_\lambda^2-\Delta m^2}{M\Delta m}G^{(T)}(k_\lambda^2) \tag{4.7}
$$

we obtain approximately [21,22]

$$
(e/M)G_2(\Delta m^2) = d - \Delta m T. \tag{4.8}
$$

Note that Δm here has pure kinematical origin, that is with $\Delta m = 0$ the decay discussed would not go. Hence, we are forced conserving the possibility of baryon decays to go over to the threshold value of $k_\lambda = \Delta m^2$ instead of the static point $k_\lambda^2 = \boldsymbol{k}^2 = 0$ where the diagonal moments are usually determined.

The last formula partly resolves a puzzle with the Hara theorem. Indeed in the $SU(3)_f$ limit:

- The dipole transition moments of the charged hyperon decays should vanish and presumably stay small due to Ademollo-Gatto theorem [13] even in the presence of the $SU(3)_f$ breaking terms;
- The toroid transition dipole moments defined by the Eq.(4.6) need not to be zero for these decays as their contributions decouples automatically in the limit $\Delta m = 0$.

So the toroid transition dipole moment of the $\Sigma^+ \to p + \gamma$ may be in the origin of the large asymmetry observed [11].

5. THE NEW VERSION OF THE HARA THEOREM

In order to state our result in another way we write the PV part of the weak radiative transition matrix element with the Lorentz structure $O_\mu^T = i\epsilon_{\mu\nu\lambda\rho} P_\nu k_\lambda \gamma_\rho$ in the framework of the $SU(3)_f$ symmetry approach following strictly [9] as

$$M = J_\mu^{(T)} \epsilon_\mu + H.C. = \{a^T(\overline{B}_3^2 O_\mu^T B_1^1 + \overline{B}_2^3 O_\mu^T B_1^1 + \overline{B}_1^1 O_\mu^T B_3^2 + \overline{B}_1^1 O_\mu^T B_2^3) + b^T(\overline{B}_1^3 O_\mu^T B_2^1 + \overline{B}_1^2 O_\mu^T B_3^1) + c^T(\overline{B}_2^1 O_\mu^T B_1^3 + \overline{B}_3^1 O_\mu^T B_1^2)\}\epsilon_\mu, \qquad (5.1)$$

where B_α^β is the $SU(3_f)$ baryon octet, $B_1^3 = p$, $B_1^2 = \Sigma^+$ etc., and a^T and b^T, c^T are up to a factor the toroid dipole moments of the neutral and charged hyperon weak radiative transitions, respectively. It is easy to see that this matrix element is invariant under the exchange of the indices 2 to 3 and *vice versa* as it should be in the standard model of weak interaction. Positive signs in front of every baryon bilinear combination arrive due to

Hermitian conjugation properties of the relevant Lorentz structure O_μ^T. Now in Eq.(5.1) all 6 PV radiative transitions are open in contrast to the Hara expression [9]:

$$M = J_\mu^{(d)}\epsilon_\mu + H.C. = a^d(\overline{B}_3^2 O_\mu^d B_1^1 + \overline{B}_2^3 O_\mu^d B_1^1 - \overline{B}_1^1 O_\mu^d B_3^2 - \overline{B}_1^1 O_\mu^d B_2^3)\epsilon_\mu, \tag{5.2}$$

based on another Lorentz structure form $O_\mu^d = i\sigma_{\mu\nu}k_\nu\gamma_5$ [9] Due to Hermitian conjugation properties of it PV transitions for the decays $\Sigma^+ \to p+\gamma$ and $\Xi^- \to \Sigma^- + \gamma$ do not appear in the Eq.(5.2) as the relevant terms

$$\overline{B}_1^3 O_\mu^d B_2^1 + \overline{B}_1^2 O_\mu^d B_3^1,$$

being invariant under exchange of indices 2 to 3 and *vice versa* , change sign under Hermitian conjugation and so sum up to zero. This is in fact a source of all the troubles with the hyperon weak radiative decays evaluations. Indeed contributions of the neutral hyperon decays $(\Sigma^0,\Lambda) \Rightarrow n+\gamma$, $\Xi^0 \to (\Sigma^0,\Lambda)+\gamma$ and those charged $\Sigma^+ \to p+\gamma$ and $\Xi^- \to \Sigma^- + \gamma$ being decoupled, it is difficult to hope that $SU(3)_f$ symmetry breaking terms would be of the same strength as the coupling constant a^d, as the Ademollo-Gatto theorem [13] forbids here great corrections. Moreover, even any $SU(3)_f$ symmetry corrections are present, they should be more or less equal for pairs $(\Sigma^0,\Lambda) \Rightarrow n+\gamma$, $\Sigma^+ \to p+\gamma$ and $\Xi_0 \to \Sigma^0+\gamma$, $\Xi^- \to \Sigma^- + \gamma$ as in both cases there are practically the same differences of masses either in charged or in neutral hyperon weak radiative decays. And then one predicts zero or small asymmetry for the decays $\Sigma^+ \to p+\gamma$ and $\Xi^- \to \Sigma^- + \gamma$ and non-zero (and eventually large) asymmetry for the decays $(\Sigma^0,\Lambda) \Rightarrow n+\gamma$ and $\Xi^0 \to (\Sigma^0,\Lambda)+\gamma$. As we know experimental data give large and negative asymmetry for the decay $\Sigma^+ \to p+\gamma$ and indicate an asymmetry of the same order of magnitude for other measurable decays in net contradiction with the Hara theorem prediction. Instead the parameters $b^T, c^T \sim T$ in Eq.(5.1) opens a possibility to account for large nonzero asymmetry in the charged hyperon radiative decays even in the $SU(3)_f$ symmetry limit and thus eventually overcome a contradiction with experiment. We display in the Table 1 the experimental data from [11] and in the Table 2 the results of Eq.(5.1) (see the 3rd column of the Table 2) and [9] (see the 2nd column of the Table 2). We

have also put there results of a traditional single-quark radiative transition which we have taken from [15] just to show in what way our new current in Eq.(5.1) can reproduce results of a quark model in a single-quark diagram approximation (that is with a single quark weak radiative transition $s \to d + \gamma$, two other quarks being spectators). For that purpose it is sufficient to assume that hyperon radiative decays in the $SU(3)_f$ model are described by the effective $|\Delta S = 1|$ octet neutral weak current similar in form to that of the effective $|\Delta S = 1|$ nonleptonic hyperon transitions (see, e.g., [30]):

$$J_\mu^W = (-F + D)\overline{B}_\gamma^3 O_\mu^T B_2^\gamma + (F + D)\overline{B}_2^\gamma O_\mu^T B_\gamma^3. \tag{5.3}$$

This *ad hoc* assumption in the usual $SU(6)$ symmetry limit (that is with $F = 2/3D$) which corresponds to use of the nonrelativistic quark model, yields the known results of the single-quark approach as can be easily seen from the Table 2 (note a sign difference for the Ξ hyperon wave function in [15] while putting $F = -2/3b, D = -b$). Instead with the $F = 0$ one arrives at the results given by the new formulation of the Hara theorem with $-a^T = b^T = c^T = D$. This indicates explicitly a source of contradiction between the Hara theorem [9] and single-quark model [15] predictions.

Assuming that all asymmetry in the decay $\Sigma^+ \to p + \gamma$ is given by the toroid dipole transition moment we have found an upper bound for it from the experimental data [11] using Eqs.(2.2, 2.3, 5.1) to be $|T| < 10^{-34}$ (cm^2). This value turns out to be close to the predicted values of the toroid dipole moments of the neutrinos ν_μ ($T_{\nu_\mu} \approx e[+1.090$ to $+2.329] \times 10^{-34}$ (cm^2)) and ν_τ ($T_{\nu_\tau} \approx e[-1.971$ to $-0.732] \times 10^{-34}$ (cm^2)) but noticeably lower than that of the ν_e neutrino ($T_{\nu_e} \approx e[+6.873$ to $8.112] \times 10^{-34}$ (cm^2)) [31].

6. NEW DERIVATION OF THE VASANTI FORMULA

Radiative hyperon decays were analyzed first in [20] at the quark level upon taking into account chiral invariance considerations. Later similar results were obtained with the Feynman diagram technique in the frame of the Weinberg-Salam model [19]. We shall try to re-derive the main result of [20], namely, that the PV single-quark weak radiative transition

$s \to d + \gamma$ is proportional to $(m_s - m_d)$. Similar to [20] we assume that quarks are on their mass shell. So at the quark level we write instead of the Vasanti formula

$$M = \frac{Ge}{\sqrt{2}} sin\theta cos\theta \overline{d} \left[a + b\gamma_5\right] i\sigma_{\mu\nu} k_\nu \epsilon_\mu s$$

for the amplitude of the $s \to d + \gamma$ decay with G, e, θ being Fermi constant, unit of charge and Cabbibo angle, respectively, $q(q = d, s)$ also meaning here spinor of the quark q with the momentum p_q, another one, using the Lorentz structure $O^T_\mu = i\epsilon_{\mu\nu\lambda\rho} P_\nu k_\lambda \gamma_\rho$, where now $P_\nu = (p_s + p_d)_\nu$, $k_\nu = (p_s - p_d)_\nu$:

$$M = \frac{Ge}{\sqrt{2}} sin\theta cos\theta \overline{d} \gamma_5 (a' + b'\gamma_5) i\epsilon_{\mu\nu\lambda\rho} P_\nu k_\lambda \gamma_\rho \epsilon_\mu \gamma_5 s, \tag{6.1}$$

and upon using generalized Gordon identities, where now all quark quantities are assumed (quarks are on their mass shell), arrive at

$$M = \frac{Ge}{\sqrt{2}} sin\theta cos\theta \overline{d} \left[a'(m_s + m_d) + b'(m_s - m_d)\gamma_5\right] i\sigma_{\mu\nu} k_\nu \epsilon_\mu s, \tag{6.2}$$

that is, in fact the main Vasanti result [20] is reproduced. The factors $(m_s \pm m_d)$ arrive due to the generalized Gordon identities. The relative signs of a' and b' are not fixed here so it is possible to obtain negative value of the asymmetry parameter. Note that Eq.(6.2) (with $a' = b' = 1$) was obtained in [20] upon assuming (i) chiral invariance, (ii) validity of the original Hara theorem. We have proved in fact that the introduction of the Lorentz structure O^T at the quark level is equivalent to the chiral invariance approach of [20] and to the diagram calculation result of [19]. This result dictates the insertion of the factor $(m_s - m_d)$ into the parameter c (see the 2nd column of the Table 2 and single-quark transition terms in [15,16] and other works cited in [12]) to assure the correct behavior of the corresponding quark PV transition amplitudes. And *vice versa*, the results of [20] and [19] together with the generalized Gordon identities show that at the quark level it is a toroid dipole moment with its characteristic Lorentz structure $O^T = i\epsilon_{\mu\nu\lambda\rho} P_\nu k_\lambda \gamma_\rho$ which is generated through the Feynman diagrams contributions of [19]. This result is valid also for the penguin diagram contributions (see, e.g., [29]) which have similar to Eq.(6.2) Lorentz structure for the case

of a real photon emission. Indeed, single-quark contributions yield the values of the toroid dipole moment T for the $s \to d + \gamma$ decay at the level of $\sim e10^{-35}\mathrm{cm}^2$, while QCD one-loop corrections (for recent calculations see, e.g., [32]) have a trend to diminish it drastically. The long-distance corrections (see, e.g. [33]) could raise it by an order of magnitude.

7. TWO-QUARK WEAK RADIATIVE TRANSITIONS

It is known that single-quark transitions $s \to d + \gamma$ give only a small part of the hyperon weak radiative decay rate. It can be seen already from the analysis of experimental data on $\Sigma^+ \to p\gamma$ and $\Xi^- \to \Sigma^-\gamma$. The decay $\Xi^- \to \Sigma^-\gamma$ can go only through a single-quark diagram and its branching ratio occurs to be at the level of 10^{-4}, while other measured decays, including the decay $\Sigma^+ \to p\gamma$ have branching ratios an order of magnitude higher, 10^{-3}. So although we have shown that a single-quark transition $s \to d + \gamma$ can be understood in terms of the toroid dipole moment it proves to be inadequate alone to describe experimental data in the framework of a quark model. Neither penguin diagram contributions are strong enough to enhance an effective $s \to d + \gamma$ [29]. This result is not surprising as the same has been proved to be true for the effective $q \to q' + \gamma$ where q, q' are light quarks [34,35]. So one has also to consider the two-quark weak radiative transitions $s + u \to u + d + \gamma$ which proceed via W-exchange and appears to be dominant. It has been done in the thoughtful works of [15,16] and others and compiled in a very complete review of [17]. In the calculations it has been assumed that all the external quarks are on their mass shell. The decay amplitudes were evaluated by sandwiching two-quark weak radiative transition operator between the baryon wave functions, one of the quarks being a spectator. We show here that the results of [15,16] can be obtained within the nonrelativistic quark model (NRQM). In this way a treatment of the hyperon weak radiative decays comes close to those of magnetic moments and weak β-decay coupling constants in the framework of NRQM [27], [28]. The results of [15] do not rely heavily on the choice of the current quarks. Indeed, if one wants to treat the quark diagrams properly one should use current quarks. But the internal symmetry of

two-quark W-exchange contributions into the hyperon radiative decays can be understood, as we shall see, already at the level of the nonrelativistic quark model with the baryon wave functions given by the $SU(6)$ model. Let us just start from the NRQM diagrams of the kind given in Fig.1 and Fig.2. We shall write here at some length the way of our reasoning for $\Sigma^+ \to p\gamma$ decay, putting other decays into Appendix A. Thus the $\Sigma^+ \to p\gamma$ decay amplitude can be casted in the form

$$\begin{aligned} 6 < p_\downarrow, \gamma(+1)|O|\Sigma^+_\uparrow >= \\ < 2u_2u_2d_1 - u_2d_2u_1 - d_2u_2u_1, \gamma(+1)|O|2u_1u_1s_2 - u_1s_1u_2 - s_1u_1u_2 >= \\ 4 < u_2u_2d_1, \gamma(+1)|O|u_1u_1s_2 > -4 < u_2u_2d_1, \gamma(+1)|O|u_1s_1u_2 > - \\ 4 < u_2d_2u_1, \gamma(+1)|O|u_1u_1s_2 > +4 < u_2d_2u_1, \gamma(+1)|O|u_1s_1u_2 >, \end{aligned} \tag{7.1}$$

where $q_{1,2}$ just mean the helicity state $q_{\uparrow,\downarrow}$ of the quarks inside the baryon, respectively. The O is an operator which we do not need to write explicitly here. The 1st matrix element on the RHS of the Eq.(7.1) $< u_2u_2d_1, \gamma(+1)|O|u_1u_1s_2 >$ in the case of W-exchange between the quarks can be described only by the diagram Fig.1(1), as there is not possible to represent it by a diagram with a spectator quark. We disregard it following the reasons of [16]. Really it is plausible that three-quark transition involving W-exchange between two quarks and a photon emission by the 3rd quark is suppressed due to kinematical reasons. Instead the 2nd matrix element on the RHS of the Eq.(7.1) $< u_2u_2d_1, \gamma(+1)|O|u_1s_1u_2 >= A_1$ can be described by three different diagrams Fig.2(1) with the quark q_2 as a spectator,

$$A_1 = \frac{2}{3}A - \frac{1}{3}E + \frac{2}{3}B.$$

Here A corresponds to the helicity non-flip weak transition amplitude with all quark heaving helicity 'up', $s_\uparrow + u_\uparrow \to u_\uparrow + d_\uparrow$, E corresponds to the helicity non-flip weak transition amplitude with different helicities of the interacting quarks $s_\downarrow + u_\uparrow \to u_\downarrow + d_\uparrow$ while B corresponds to the helicity-flip weak transition amplitude with the 'up' helicity of the s quark $s_\uparrow + u_\downarrow \to u_\downarrow + d_\uparrow$, the factors 2/3 and -1/3 are just the values of the quark charges in term of the proton electric charge e. The 3nd matrix element on the RHS of the Eq.(7.1)

$< u_2 d_2 u_1, \gamma(+1)|O|u_1 u_1 s_2 >= A_3$ can be described by three different diagrams Fig.2(3) with the quark q_1 as a spectator,

$$A_3 = \frac{2}{3}C - \frac{1}{3}E + \frac{2}{3}\tilde{A},$$

with two new quantities, C corresponding to the helicity-flip weak transition amplitude with the 'down' helicity of the s quark, $s_\downarrow + u_\uparrow \to u_\uparrow + d_\downarrow$, and $\tilde{A}$ corresponding to the helicity non-flip weak transition amplitude with down helicities of the interacting quarks, $s_\downarrow + u_\downarrow \to u_\downarrow + d_\downarrow$. The 4nd matrix element on the RHS of the Eq.(7.1) $< u_2 d_2 u_1, \gamma(+1)|O|u_1 s_1 u_2 >$ is described by two sets of diagrams, these given by the Fig.2(2) with quark q_2 as a spectator,

$$A_2 = -\frac{1}{3}A - \frac{1}{3}C + \frac{2}{3}D,$$

where a new quantity D corresponds to the helicity non-flip weak transition amplitude with different helicities of the interacting quarks, $s_\uparrow + u_\downarrow \to u_\uparrow + d_\downarrow$, and those given by the Fig.2(4) with the quark q_1 as a spectator,

$$A_4 = \frac{2}{3}D - \frac{1}{3}\tilde{A} - \frac{1}{3}B,$$

So finally

$$< p_\downarrow, \gamma(+1)|O|\Sigma^+_\uparrow >= \frac{2}{3}(-2A_1 + A_2 - 2A_3 + A_4) \tag{7.2}$$

If we assume that a spectator quark does not induce changes in the A_k, $k = 1, 2, 3, 4$, in this approximation all the hyperon radiative decays can be written in terms of the quantities $A_{1,2,3,4}$:

$$\begin{aligned}
< p_\downarrow, \gamma(+1)|O|\Sigma^+_\uparrow >&= \frac{2}{3}(-2A_1 + A_2 - 2A_3 + A_4),\\
< n_\downarrow, \gamma(+1)|O|\Sigma^0_\uparrow >&= \frac{2}{3\sqrt{2}}(A_1 - 2A_2 - 2A_3 + A_4),\\
< n_\downarrow, \gamma(+1)|O|\Lambda_\uparrow >&= \frac{2}{\sqrt{6}}(A_1 - 2A_2 - A_4),\\
< \Lambda_\downarrow, \gamma(+1)|O|\Xi^0_\uparrow >&= \frac{2}{\sqrt{6}}(A_1 - A_2),\\
< \Sigma^0_\downarrow, \gamma(+1)|O|\Xi^0_\uparrow >&= \frac{2}{3\sqrt{2}}(A_1 + A_2 - 2A_3 + 4A_4).
\end{aligned} \tag{7.3}$$

Two-quark W-exchange amplitudes satisfy two relations:

$$< p_\downarrow, \gamma(+1)|O|\Sigma^+_\uparrow > +2\sqrt{6} < \Lambda_\downarrow, \gamma(+1)|O|\Xi^0_\uparrow >=$$

$$\sqrt{2} < \Sigma^0_\downarrow, \gamma(+1)|O|\Xi^0_\uparrow > +\sqrt{6} < n_\downarrow, \gamma(+1)|O|\Lambda_\uparrow >,$$

$$\sqrt{2} < n_\downarrow, \gamma(+1)|O|\Sigma^0_\uparrow >=< p_\downarrow, \gamma(+1)|O|\Sigma^+_\uparrow > +\sqrt{6} < \Lambda_\downarrow, \gamma(+1)|O|\Xi^0_\uparrow >,$$

as they depend not on all A_k's, but only on their linear combinations $A_1 - A_2$, $A_2 + A_3$, $A_3 - A_4$. It is straightforward to show that with

$$\begin{aligned}
A_1^{PV} &= \frac{1}{6} + \frac{2}{3}X - \zeta(\frac{1}{3} + \frac{4}{3}X), \quad A_2^{PV} = -\frac{1}{6} + \frac{2}{3}X - \zeta(-\frac{1}{3} + \frac{4}{3}X), \\
A_3^{PV} &= \frac{1}{6} - \frac{1}{3}X - \zeta X, \quad A_4^{PV} = -\frac{1}{6} - \frac{2}{3}X - \zeta X, \\
A_1^{PC} &= \frac{1}{6} + \frac{2}{3}X - \zeta(\frac{1}{3} + \frac{4}{3}X), \quad A_2^{PC} = -\frac{1}{6} + \frac{2}{3}X - \zeta(-\frac{1}{3} + \frac{4}{3}X), \\
A_3^{PC} &= -\frac{1}{6} - \frac{2}{3}X - \zeta X, \quad A_4^{PC} = \frac{1}{6} - \frac{1}{3}X - \zeta X,
\end{aligned} \tag{7.4}$$

where $X = k/2m_u$ and $6\zeta = (1 - m_u/m_s)$ [15], one arrives up to an overall constant exactly at the results of [15] (See Table 1 in [15] and the 3rd column of the Table 3 of this work) and with $X = 0$, $\epsilon = 1 - 6\zeta = 0$ we return to the results of [17].

So the main results of the two-quark weak radiative transition model can be understood in the framework of NRQM in terms of the quantities A_k, $k = 1, 2, 3, 4,$, which however cannot be calculated without further assumptions. Neither we have answered the question where is the source of a contradiction between the two-quark transition model results and those of Hara [9]. In order to answer at least partly to this question we shall proceed in a way similar to that of the [15] and [36], returning to current quarks within the Salam-Weinberg model. Let us consider kinematics of one of the two-quark PV weak radiative transition in some detail. PV part of the Feynman diagram for the bremsstrahlung process $s + u \to u + d + \gamma$ with the 3rd quark q as a spectator, where γ-quantum irradiates from the u-quark, reads [15] (we write $q(p_k), q = u, d, s, k = 1, \ldots 6$, for a spinor of a quark q with momentum p_k, while (ab) means below a scalar product $a_\mu b_\mu$):

$$
\begin{aligned}
&< u(p_2), d(p_4), q(p_6), \gamma(k)|O^{PV}|s(p_1), u(p_3), q(p_5) >= \\
&e_u/(p_2k)\overline{u}(p_2)\hat{k}\hat{\epsilon}(k)\gamma_\mu\gamma_5 s(p_1)\overline{d}(p_4)\gamma_\mu u(p_3)\overline{q}(p_6)q(p_5) + \\
&\quad e_u/(p_2k)\overline{u}(p_2)\hat{k}\hat{\epsilon}(k)\gamma_\mu s(p_1)\overline{d}(p_4)\gamma_\mu\gamma_5 u(p_3)\overline{q}(p_6)q(p_5).
\end{aligned}
\tag{7.5}
$$

Starting from similar expression, two-quark diagrams were calculated in the Coulomb gauge [15], upon carrying out an expansion of the amplitude given by the Fig.1(1) in photon momentum k. (Recently with another technique similar calculations have been performed for the bremsstrahlung process $b+u \to u+s+\gamma$ [36].) Instead we use an identity (see, e.g., [28]) for the 1st term in the RHS of the Eq.(7.5)

$$
\gamma_\alpha\gamma_\beta\gamma_\rho = (\delta_{\alpha\beta}\delta_{\rho\delta} - \delta_{\alpha\rho}\delta_{\beta\delta} + \delta_{\alpha\delta}\delta_{\beta\rho})\gamma_\delta - \epsilon_{\alpha\beta\rho\delta}\gamma_\delta\gamma_5
$$

in order to rewrite it (up to a factor $e_u/(p_2k)$) in the form

$$
\begin{aligned}
&k_\alpha\epsilon_\beta(k)\overline{u}(p_2)\gamma_\alpha\gamma_\beta\gamma_\mu\gamma_5 s(p_1)V_\mu\overline{q}(p_6)q(p_5) = \overline{u}(p_2)\{(k\epsilon)\gamma_\mu\gamma_5 s(p_1)V_\mu - \\
&\quad \hat{k}\gamma_5 s(p_1)(V\epsilon) + \hat{\epsilon}\gamma_5 s(p_1)(Vk) - \epsilon_{\alpha\beta\mu\delta}k_\alpha\epsilon_\beta(k)\gamma_\delta s(p_1)V_\mu\}\overline{q}(p_6)q(p_5),
\end{aligned}
\tag{7.6}
$$

where

$$
V_\mu = \overline{d}(p_4)\gamma_\mu u(p_3).
$$

Upon using Gordon identity, assuming the equality of the masses of quarks d and u and of their momenta, we obtain that

$$
V_\mu = \frac{1}{2m_{u,d}} p_{4\mu}\overline{d}(p_4)u(p_3).
$$

that is, the vector V_μ is proportional to the momenta of quarks $u(p_3)$ and/or $d(p_4)$. In a nonrelativistic quark model it is reasonable to assume at least for the hyperon decays $\Sigma^+ \to p\gamma$ and $(\Sigma^0, \Lambda) \to n\gamma$ that the momenta of the quarks of the final nucleon are equal, that is $p_2 = p_4 = p_6$, and also for the spectator quark the equality $p_5 = p_6$ holds. In this oversimplified picture kinematics of the hyperon decay is related to the kinematics of the quarks as $k = P_i - P_f = p_1 - p_2$, $\quad P = P_i + P_f = p_1 + 5p_2$ and $p_4 = \frac{1}{6}(P-k)$. The last term in the RHS of the Eq.(7.6) then gives the structure $\epsilon_{\alpha\beta\mu\delta}k_\alpha\epsilon_\beta P_\mu\gamma_\delta$ characteristic for the

toroid dipole transition. This explains explicitly why the quark model calculations of the two-quark contributions into the weak radiative transition $\Sigma^+ \to p\gamma$ do not follow the Hara theorem. The important part of their contribution has a Lorentz structure different from that used in the Hara theorem and with another properties under Hermitian conjugation. An estimation of the second term in the RHS of the Eq.(7.5) shows that its contribution does not change our result. The same is true for other four hyperon weak radiative decays $(\Sigma^0, \Lambda) \Rightarrow n + \gamma$ and $\Xi^0 \to (\Sigma^0, \Lambda) + \gamma$. Note that we have obtained charged axial-vector current multiplied by the charged scalar current (and by the neutral scalar current of the spectator quark). But our conclusion for the PV transition amplitude as a whole remains the same while Fierz rearrangement would allow to obtain an expression similar to that of the single-quark transition given by Eq.(6.1) and consequently to arrive at the description of the radiative transition as a whole in terms of hyperons (cf. Eq.(5.1)). We shall write it in more detail elsewhere.

8. SUMMARY AND CONCLUSION

In order to resolve a contradiction between the experiments claiming large negative asymmetry in $\Sigma^+ \to p + \gamma$, the Hara theorem, predicting zero asymmetry for $\Sigma^+ \to p + \gamma$ and $\Xi^- \to \Sigma^- + \gamma$ in the exact $SU(3)_f$ symmetry and quark models which cannot reproduce the Hara theorem results without making vanish all asymmetry parameters in the $SU(3)_f$ symmetry limit, we have considered a parity-violating part of the transition electromagnetic current in the alternative form allowing well-defined multipole expansion. Part of it which is connected with the Lorentz structure $i\epsilon_{\mu\nu\lambda\rho}P_\nu k_\lambda \gamma_\rho$ enables as to reformulate the Hara theorem thus opening a possibility of nonzero asymmetry parameters for all 6 weak radiative hyperon decays and revealing hitherto unseen transition toroid dipole moments. In this way at least partly it is resolved a long-stayed puzzle with Hara theorem prediction and experimental result for the asymmetry of the weak radiative decay $\Sigma^+ \to p + \gamma$. We have also reproduced Vasanti formula at the quark level. Our result is consistent with the

traditional results of the single-quark transition models and is unaltered by the QCD corrections including the penguin diagram contributions. As to the two-quark weak radiative transitions we have found that the main part of the diagram contribution is also connected with the Lorentz structure $i\epsilon_{\mu\nu\lambda\rho}P_\nu k_\lambda\gamma_\rho$ which seems to be an intrinsic reason of the apparent contradiction between the Hara theorem conclusion and quark model results for hyperon weak radiative decays.

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APPENDIX A

Two – quark W – exchange diagrams in the NRQM

(i) The decay $\Sigma^+ \to p\gamma$ is described in the main text.

(ii) The next one is though unseen but important for model reason the $\Sigma^0 \to n\gamma$ decay.

$$6\sqrt{2} < n_\downarrow, \gamma(+1)|O|\Sigma^0_\uparrow >=$$

$$< 2d_2d_2u_1 - d_2u_2d_1 - u_2d_2d_1, \gamma(+1)|O|2u_1d_1s_2 + 2d_1u_1s_2 -$$

$$u_1s_1d_2 - s_1u_1d_2 - d_1s_1u_2 - s_1d_1u_2 >=$$

$$8 < d_2d_2u_1, \gamma(+1)|O|u_1d_1s_2 > -8 < d_2d_2u_1, \gamma(+1)|O|u_1s_1d_2 > - \qquad (A.1)$$

$$4 < d_2d_2u_1, \gamma(+1)|O|d_1s_1u_2 > -8 < d_2u_2d_1, \gamma(+1)|O|u_1d_1s_2 > +$$

$$4 < d_2u_2d_1, \gamma(+1)|O|u_1s_1d_2 > +4 < d_2u_2d_1, \gamma(+1)|O|d_1s_1u_2 >$$

The 1st and the 3rd matrix elements on the RHS of the Eq.(A.1) in the case of W-exchange between the quarks can be described only by the diagrams Fig.1(2) and Fig.1(3), and we disregard them following, as its cannot be represented by diagrams with a spectator quark. The 2nd matrix element on the RHS of the Eq.(A.1) $< d_2d_2u_1, \gamma(+1)|O|u_1s_1d_2 >= A_2$ can be described by three diagrams Fig.2(2) with the quark d_2 as a spectator. The 4nd matrix element on the RHS of the Eq.(A.1) $< d_2u_2d_1, \gamma(+1)|O|u_1d_1s_2 >= A_3$ can be described by three diagrams Fig.2(3) with the quark d_1 as a spectator. The 5th matrix element on the RHS of the Eq.(A.1) is $< d_2u_2d_1, \gamma(+1)|O|u_1s_1d_2 >= A_1$ given by three diagrams of the Fig.2(1) with the quark d_2 as a spectator. The 6th matrix element on the RHS of the Eq.(A.1) is $< d_2u_2d_1, \gamma(+1)|O|d_1s_1u_2 >= A_4$ given by three diagrams of the Fig.2(4) with the quark d_1 as a spectator. So

$$< n_\downarrow, \gamma(+1)|O|\Sigma^0_\uparrow >= \frac{2}{3\sqrt{2}}(A_1 - 2A_2 - 2A_3 + A_4).$$

(iii) The $\Lambda \to n\gamma$ decay amplitude can be written in the form

$$2\sqrt{6} < n_\downarrow, \gamma(+1)|O|\Lambda_\uparrow >=$$

$$< 2d_2d_2u_1 - d_2u_2d_1 - u_2d_2d_1, \gamma(+1)|O|u_1s_1d_2 + s_1u_1d_2 - d_1s_1u_2 - s_1d_1u_2 >=$$

$$4 < d_2d_2u_1, \gamma(+1)|O|u_1s_1d_2 > -4 < d_2d_2u_1, \gamma(+1)|O|d_1s_1u_2 > - \qquad (A.2)$$

$$4 < d_2u_2d_1, \gamma(+1)|O|u_1s_1d_2 > +4 < u_2d_2d_1, \gamma(+1)|O|d_1s_1u_2 > .$$

The 1st matrix element on the RHS of the Eq.(A.2) $< d_2d_2u_1, \gamma(+1)|O|u_1s_1d_2 >= A_2$ can be described by three diagrams Fig.2(2) with the quark d_2 as a spectator. The 2nd matrix element on the RHS of the Eq.(A.2) $< d_2d_2u_1, \gamma(+1)|O|d_1s_1u_2 >$ in the case of W-exchange between the quarks can be described only by the diagram Fig.1(3), as it cannot be represented by a diagram with a spectator quark. The 3rd matrix element on the RHS of the Eq.(A.2) $< d_2u_2d_1, \gamma(+1)|O|u_1s_1d_2 >= A_1$ can be described by three diagrams Fig.2(1) with the quark d_2 as a spectator. The 4th matrix element on the RHS of the Eq.(A.2) $< d_2u_2d_1, \gamma(+1)|O|d_1s_1u_2 >= A_4$ can be described by three diagrams Fig.2(4) with the quark d_1 as a spectator. So

$$< n_\downarrow, \gamma(+1)|O|\Lambda_\uparrow >= \frac{2}{\sqrt{6}}(A_1 - 2A_2 - A_4).$$

(iv) The $\Xi^0 \to \Lambda\gamma$ decay amplitude can be written in the form

$$2\sqrt{6} < \Lambda_\downarrow, \gamma(+1)|O|\Xi^0_\uparrow >=$$

$$< u_2s_2d_1 + s_2u_2d_1 - d_2s_2u_1 - s_2d_2u_1, \gamma(+1)|O|2s_1s_1u_2 - s_1u_1s_2 - u_1s_1s_2 >=$$

$$4 < u_2s_2d_1, \gamma(+1)|O|s_1s_1u_2 > -4 < u_2s_2d_1, \gamma(+1)|O|s_1u_1s_2 > - \qquad (A.3)$$

$$4 < d_2s_2u_1, \gamma(+1)|O|s_1s_1u_2 > +4 < d_2s_2u_1, \gamma(+1)|O|s_1u_1s_2 > .$$

The 1st and 3rd matrix elements on the RHS of Eq.(A.3) in the case of W-exchange between the quarks can be described only by the diagrams Fig.1(4) and Fig.1(5), and we disregard them, as they cannot be represented by diagrams with a spectator quark.

The 2nd matrix element on the RHS of the Eq.(A.3) $< u_2 s_2 d_1, \gamma(+1)|O|s_1 u_1 s_2 >= A_1$ can be described by three diagrams Fig.2(1) with the quark s_2 as a spectator. The 4th matrix element on the RHS of the Eq.(A.3) $< d2 s_2 u_1, \gamma(+1)|O|s_1 u_1 s_2 >= A_2$ can be described by three diagrams Fig.2(2) with the quark s_2 as a spectator. So

$$< \Lambda_\downarrow, \gamma(+1)|O|\Xi^0_\uparrow >= \frac{2}{\sqrt{6}}(A_1 - A_2).$$

(v) The $\Xi^0 \to \Lambda\gamma$ decay amplitude can be written in the form

$$6\sqrt{2} < \Sigma^0_\downarrow, \gamma(+1)|O|\Xi^0_\uparrow >=< 2u_2 d_2 s_1 + 2d_2 u_2 s_1$$

$$-u_2 s_2 d_1 - s_2 u_2 d_1 - d_2 s_2 u_1 - s_2 d_2 u_1, \gamma(+1)|O|2s_1 s_1 u_2 - s_1 u_1 s_2 - u_1 s_1 s_2 >=$$

$$8 < u_2 d_2 s_1, \gamma(+1)|O|s_1 s_1 u_2 > -8 < u_2 d_2 s_1, \gamma(+1)|O|s_1 u_1 s_2 > - \qquad (A.4)$$

$$4 < u_2 s_2 d_1, \gamma(+1)|O|s_1 s_1 u_2 > +4 < u_2 s_2 d_1, \gamma(+1)|O|s_1 u_1 s_2 > -$$

$$4 < d_2 s_2 u_1, \gamma(+1)|O|s_1 s_1 u_2 > +4 < d_2 s_2 u_1, \gamma(+1)|O|s_1 u_1 s_2 > .$$

The 1st matrix element on the RHS of the Eq.(A.4) $< u_2 d_2 s_1, \gamma(+1)|O|s_1 s_1 u_2 >= A_4$ can be described by three diagrams Fig.2(4) with the quark s_1 as a spectator. The 2nd matrix element on the RHS of the Eq.(A.4) $< u_2 d_2 s_1, \gamma(+1)|O|s_1 u_1 s_2 >= A_3$ can be described by three diagrams Fig.2(3) with the quark s_1 as a spectator. The 3rd and 5th matrix elements on the RHS of Eq.(A.4) in the case of W-exchange between the quarks can be described only by the diagrams Fig.1(4) and Fig.1(5), and we disregard them, as they cannot be represented by diagrams with a spectator quark. The 4th matrix element on the RHS of Eq.(A.4) $< u_2 s_2 d_1, \gamma(+1)|O|s_1 u_1 s_2 >= A_1$ can be described by three diagrams Fig.2(1) with the quark s_2 as a spectator. Finally, the 6th matrix element on the RHS of the Eq.(A.4) $< d_2 s_2 u_1, \gamma(+1)|O|s_1 u_1 s_2 >= A_2$ can be described by three diagrams Fig.2(2) with the quark s_2 as a spectator. So

$$< \Sigma^0_\downarrow, \gamma(+1)|O|\Xi^0_\uparrow >= \frac{2}{3\sqrt{2}}(A_1 + A_2 - 2A_3 + 4A_4).$$

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Table 1. Hyperon weak radiative transitions, experiment [11]

Decay	BR ($\times 10^3$)	Asymmetry
$\Sigma^+ \to p\gamma$	1.23 ± 0.06	-0.76 ± 0.08
$\Sigma^0 \to n\gamma$	$-$	$-$
$\Lambda^0 \to n\gamma$	1.63 ± 0.14	$-$
$\Xi^0 \to \Lambda\gamma$	1.06 ± 0.16	$+0.44 \pm 0.44$
$\Xi^0 \to \Sigma^0\gamma$	3.56 ± 0.43	$+0.20 \pm 0.32$
$\Xi^- \to \Sigma^-\gamma$	0.128 ± 0.023	$+1.0 \pm 1.3$

Table 2. Hyperon weak radiative PV transitions, theory, $SU(3)_f$ model and single-quark diagram contributions

Decay	in [15]	in [9]	from Eqs.(5.1)	from Eq.(5.3)
$\Sigma^+ \to p\gamma$	$-b/3$	0	c^T	$-F + D$
$\Sigma^0 \to n\gamma$	$b/3\sqrt{2}$	$a^d/\sqrt{2}$	$a^T/\sqrt{2}$	$(F - D)/\sqrt{2}$
$\Lambda^0 \to n\gamma$	$3b/\sqrt{6}$	$a^d/\sqrt{6}$	$a^T/\sqrt{6}$	$-(3F + D)/\sqrt{6}$
$\Xi^0 \to \Lambda\gamma$	$b/\sqrt{6}$	$-a^d/\sqrt{6}$	$a^T/\sqrt{6}$	$(3F - D)/\sqrt{6}$
$\Xi^0 \to \Sigma^0\gamma$	$-5b/3\sqrt{2}$	$-a^d/\sqrt{2}$	$a^T/\sqrt{2}$	$-(F + D)/\sqrt{2}$
$\Xi^- \to \Sigma^-\gamma$	$5b/3$	0	b^T	$F + D$

Table 3. Hyperon weak PV radiative transitions, theory, 2-quark diagram contributions

Decay	in [17]	in [15]	from Eqs.(7.3)
$\Sigma^+ \to p\gamma$	$-\frac{5+\epsilon}{9\sqrt{2}}b$	$\frac{2}{9}[-3 - 2X + \zeta(3 + X)]$	$\frac{2}{3}(-2A_1 + A_2 - 2A_3 + A_4)$
$\Sigma^0 \to n\gamma$	$-\frac{1-\epsilon}{18}b$	$\frac{2}{9\sqrt{2}}[-2X + \zeta(-3 + X)]$	$\frac{2}{3\sqrt{2}}(A_1 - 2A_2 - 2A_3 + A_4)$
$\Lambda^0 \to n\gamma$	$\frac{3+\epsilon}{6\sqrt{3}}b$	$\frac{2}{3\sqrt{6}}[-2 + \zeta(-3 + X)]$	$\frac{2}{\sqrt{6}}(A_1 - 2A_2 - A_4)$
$\Xi^0 \to \Lambda\gamma$	$-\frac{2+\epsilon}{9\sqrt{3}}b$	$\frac{2}{3\sqrt{6}}[1 - 2\zeta]$	$\frac{2}{\sqrt{6}}(A_1 - A_2)$
$\Xi^0 \to \Sigma^0\gamma$	$\frac{1}{3}b$	$\frac{2}{9\sqrt{2}}[-3 - 2X - 2\zeta X]$	$\frac{2}{3\sqrt{2}}(A_1 + A_2 - 2A_3 + 4A_4)$
$\Xi^- \to \Sigma^-\gamma$	0	0	0

FIGURES

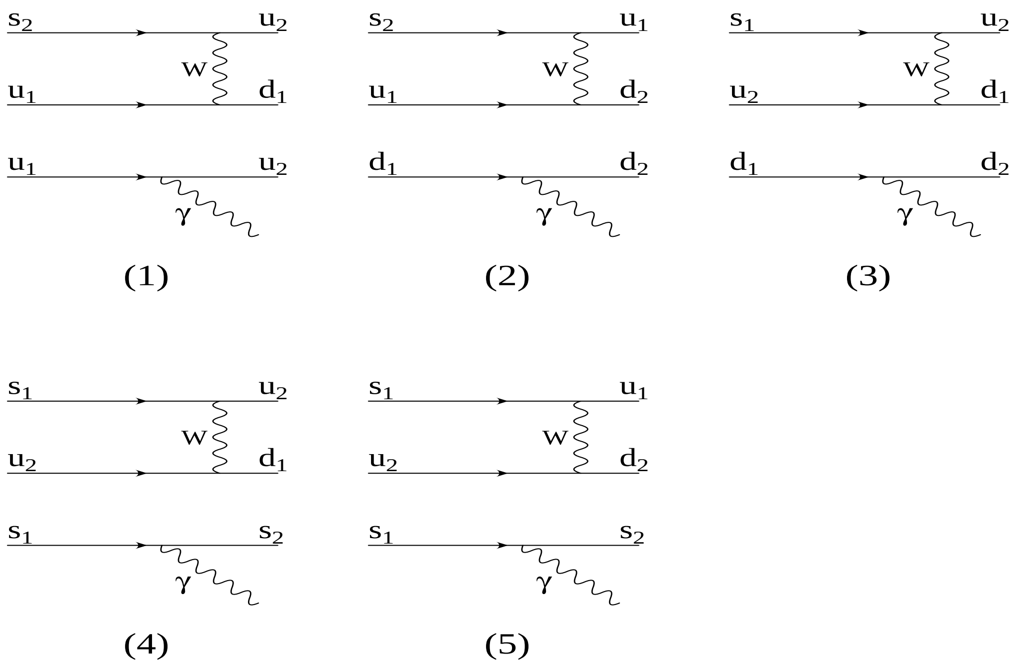

FIG. 1. Three-quark diagrams without spectator quark (q_1 means $q_\uparrow$, q_2 means $q_\downarrow$, $q = u, d, s$)

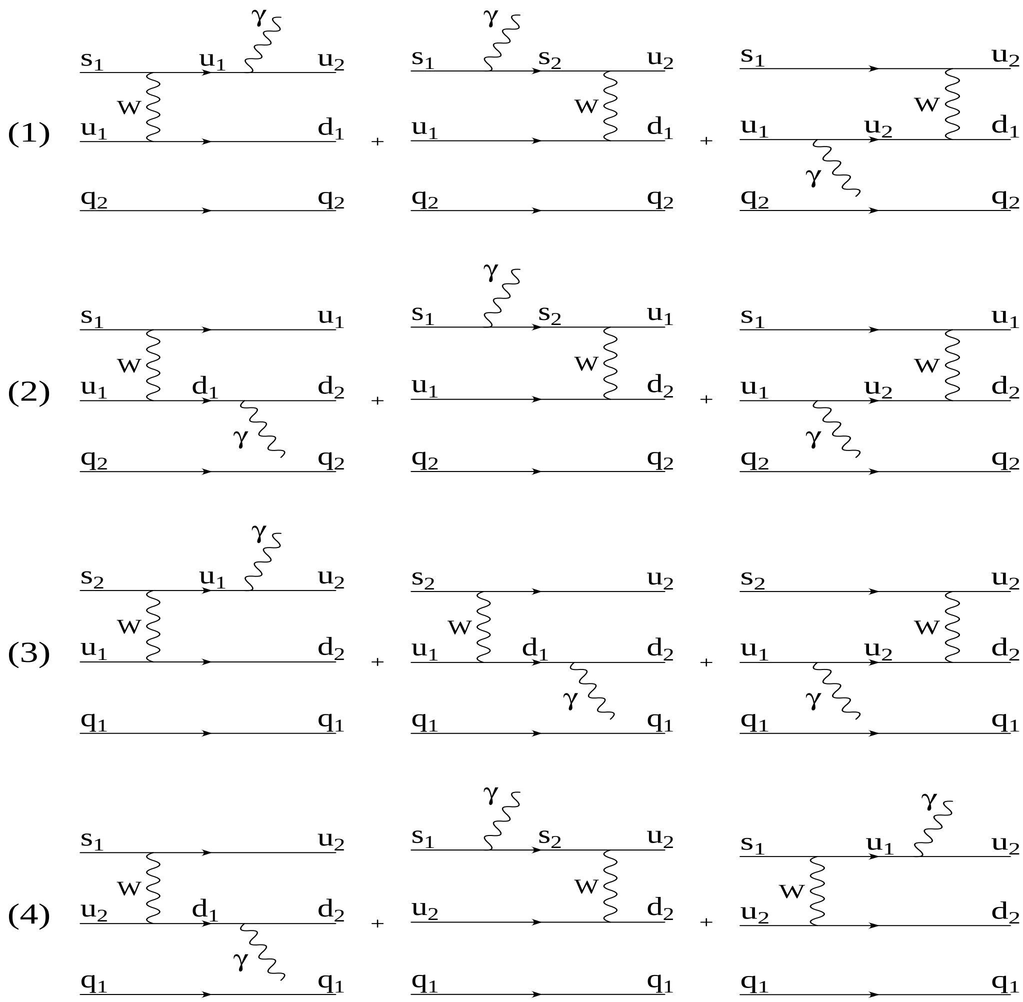

FIG. 2. Three-quark diagrams with the third quark $q_{1,2}$ as a spectator (q_1 means $q_\uparrow$, q_2 means $q_\downarrow$, $q = u, d, s$)